

## **EXHIBIT K**

## **OMNIBUS BROWN DECLARATION**

---

# A Guide to Modern Econometrics

---

2nd edition

**Marno Verbeek**

*Erasmus University Rotterdam*



John Wiley & Sons, Ltd

Copyright © 2004

John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester,  
West Sussex PO19 8SQ, England

Telephone (+44) 1243 779777

Email (for orders and customer service enquiries): [cs-books@wiley.co.uk](mailto:cs-books@wiley.co.uk)  
Visit our Home Page on [www.wileyeurope.com](http://www.wileyeurope.com) or [www.wiley.com](http://www.wiley.com)

All Rights Reserved. No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning or otherwise, except under the terms of the Copyright, Designs and Patents Act 1988 or under the terms of a licence issued by the Copyright Licensing Agency Ltd, 90 Tottenham Court Road, London W1T 4LP, UK, without the permission in writing of the Publisher. Requests to the Publisher should be addressed to the Permissions Department, John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex PO19 8SQ, England, or emailed to [permreq@wiley.co.uk](mailto:permreq@wiley.co.uk), or faxed to (+44) 1243 770620.

This publication is designed to provide accurate and authoritative information in regard to the subject matter covered. It is sold on the understanding that the Publisher is not engaged in rendering professional services. If professional advice or other expert assistance is required, the services of a competent professional should be sought.

#### ***Other Wiley Editorial Offices***

John Wiley & Sons Inc., 111 River Street, Hoboken, NJ 07030, USA

Jossey-Bass, 989 Market Street, San Francisco, CA 94103-1741, USA

Wiley-VCH Verlag GmbH, Boschstr. 12, D-69469 Weinheim, Germany

John Wiley & Sons Australia Ltd, 33 Park Road, Milton, Queensland 4064, Australia

John Wiley & Sons (Asia) Pte Ltd, 2 Clementi Loop #02-01, Jin Xing Distripark, Singapore 129809

John Wiley & Sons Canada Ltd, 22 Worcester Road, Etobicoke, Ontario, Canada M9W 1L1

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print may not be available in electronic books.

#### ***Library of Congress Cataloging-in-Publication Data***

Verbeek, Marno.

A guide to modern econometrics / Marno Verbeek. – 2nd ed.

p. cm.

Includes bibliographical references and index.

ISBN 0-470-85773-0 (pbk. : alk. paper)

1. Econometrics. 2. Regression analysis. I. Title.

HB139.V465 2004

330'.01'5195 – dc22

2004004222

#### ***British Library Cataloguing in Publication Data***

A catalogue record for this book is available from the British Library

ISBN 0-470-85773-0

Typeset in 10/12pt Times by Laserwords Private Limited, Chennai, India

Printed and bound in Great Britain by TJ International, Padstow, Cornwall

This book is printed on acid-free paper responsibly manufactured from sustainable forestry in which at least two trees are planted for each one used for paper production.

is a specific value chosen by the researcher. If this hypothesis is true we know that the statistic

$$t_k = \frac{b_k - \beta_k^0}{\text{se}(b_k)} \quad (2.49)$$

has a  $t$  distribution with  $N - K$  degrees of freedom. If the null hypothesis is not true, the alternative hypothesis  $H_1: \beta_k \neq \beta_k^0$  holds. As there are no unknown values in  $t_k$ , it becomes a **test statistic** that can be computed from the estimate  $b_k$  and its standard error  $\text{se}(b_k)$ . The usual testing strategy is to reject the null hypothesis if  $t_k$  realizes a value that is very unlikely if the null hypothesis is true. In this case this means very large absolute values for  $t_k$ . To be precise, one rejects the null hypothesis if the probability of observing a value of  $|t_k|$  or larger is smaller than a given **significance level**  $\alpha$ , often 5%. From this, one can define the **critical values**  $t_{N-K;\alpha/2}$  using

$$P\{|t_k| > t_{N-K;\alpha/2}\} = \alpha.$$

For  $N - K$  not too small, these critical values are only slightly larger than those of the standard normal distribution, for which the two-tailed critical value for  $\alpha = 0.05$  is 1.96. Consequently, at the 5% level the null hypothesis will be rejected if

$$|t_k| > 1.96.$$

The above test is referred to as a **two-sided test** because the alternative hypothesis allows for values of  $\beta_k$  on both sides of  $\beta_k^0$ . Occasionally, the alternative hypothesis is one-sided, for example: the expected wage for a man is larger than that for a woman. Formally, we define the null hypothesis as  $H_0: \beta_k \leq \beta_k^0$  with alternative  $H_1: \beta_k > \beta_k^0$ . Next we consider the distribution of the test statistic  $t_k$  at the boundary of the null hypothesis (i.e. under  $\beta_k = \beta_k^0$ , as before) and we reject the null hypothesis if  $t_k$  is too large (note that large values for  $b_k$  lead to large values for  $t_k$ ). Large negative values for  $t_k$  are compatible with the null hypothesis and do not lead to its rejection. Thus for this **one-sided test**, the critical value is determined from

$$P\{t_k > t_{N-K;\alpha}\} = \alpha.$$

Using the standard normal approximation again, we reject the null hypothesis at the 5% level if

$$t_k > 1.64.$$

Regression packages typically report the following  $t$ -value,

$$t_k = \frac{b_k}{\text{se}(b_k)},$$

sometimes referred to as the  $t$ -ratio, which is the point estimate divided by its standard error. The  $t$ -ratio is the  $t$ -statistic one would compute to test the null hypothesis that  $\beta_k = 0$ , which may be a hypothesis that is of economic interest as well. If it is rejected, it is said that ‘ $b_k$  differs significantly from zero’, or that the corresponding variable

### 2.5.7 Size, Power and p-Values

When an hypothesis is statistically tested two types of errors can be made. The first one is that we reject the null hypothesis while it is actually true, and is referred to as a **type I error**. The second one, a **type II error**, is that the null hypothesis is not rejected while the alternative is true. The probability of a type I error is directly controlled by the researcher through his choice of the significance level  $\alpha$ . When a test is performed at the 5% level, the probability of rejecting the null hypothesis while it is true is 5%. This probability (significance level) is often referred to as the **size** of the test. The probability of a type II error depends upon the true parameter values. Intuitively, if the truth deviates much from the stated null hypothesis, the probability of such an error will be relatively small, while it will be quite large if the null hypothesis is close to the truth. The reverse probability, that is, the probability of rejecting the null hypothesis when it is false, is known as the **power** of the test. It indicates how ‘powerful’ a test is in finding deviations from the null hypothesis (depending upon the true parameter value). In general, reducing the size of a test will decrease its power, so that there is a trade-off between type I and type II errors.

Suppose that we are testing the hypothesis that  $\beta_2 = 0$ , while its true value is in fact 0.1. It is clear that the probability that we reject the null hypothesis depends upon the standard error of our OLS estimator  $b_2$  and thus, among other things, upon the sample size. The larger the sample the smaller the standard error and the more likely we are to reject. This implies that type II errors become increasingly unlikely if we have large samples. To compensate for this, researchers typically reduce the probability of type I errors (that is of incorrectly rejecting the null hypothesis) by lowering the size  $\alpha$  of their tests. This explains why in large samples it is more appropriate to choose a size of 1% or less rather than the ‘traditional’ 5%. Similarly, in very small samples we may prefer to work with a significance level of 10%.

Commonly, the null hypothesis that is chosen is assumed to be true unless there is convincing evidence of the contrary. This suggests that if a test does not reject, for whatever reason, we stick to the null hypothesis. This view is not completely appropriate. A range of alternative hypotheses could be tested (for example  $\beta_2 = 0$ ,  $\beta_2 = 0.1$  and  $\beta_2 = 0.5$ ), with the result that none of them is rejected. Obviously, concluding that these three null hypotheses are simultaneously true would be ridiculous. The only appropriate conclusion is that we *cannot reject* that  $\beta_2$  is 0, nor that it is 0.1 or 0.5. Sometimes, econometric tests are simply not very powerful and very large sample sizes are needed to reject a given hypothesis.

A final probability that plays a role in statistical tests is usually referred to as the **p-value**. This  $p$  or probability value denotes the minimum size for which the null hypothesis would still be rejected. It is defined as the probability, under the null, to find a test statistic that (in absolute value) exceeds the value of the statistic that is computed from the sample. If the  $p$ -value is smaller than the significance level  $\alpha$ , the null hypothesis is rejected. Many modern software packages supply such  $p$ -values and in this way allow researchers to draw their conclusions without consulting or computing the appropriate critical values. It also shows the sensitivity of the decision to reject the null hypothesis, with respect to the choice of significance level. For example, a  $p$ -value of 0.08 indicates that the null hypothesis is rejected at the 10% significance level, but not at the 5% level.